

# A depth-first implementation of Geometric Logic in Prolog

(with proof objects)

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# GL as a fragment of FOL

- Geometric formula:  $C \Rightarrow D$
- $C = A_1 \wedge \dots \wedge A_n$  ( $n \geq 0$ ,  $A_i$  atoms)
- $D = E_1 \vee \dots \vee E_m$  ( $m \geq 0$ )
- $E_j = (E \ x_1 \dots x_k) C_j$  ( $k \geq 0$ ,  $C_j$  like  $C$ )
- Implicit universal closure
- No function symbols (yet)

# Examples

- Lattices (meet is associative):

$$x \cap y = u \wedge u \cap z = v \wedge y \cap z = w \Rightarrow x \cap w = u$$

- Projective unicity:

$$p|l \wedge p|m \wedge q|l \wedge q|m \Rightarrow p=q \vee l=m$$

- Diamond property:

$$a \rightarrow b \wedge a \rightarrow c \Rightarrow (\exists d) (b \rightarrow d \wedge c \rightarrow d)$$

# Rationale

- Less skolemization
- Direct proofs
- Constructive logic
- Natural proof theory/objects

# Inductive definition of $X \vdash_{(T)} D$

- (base)  $X \vdash D$  if  $X \downarrow D$
- (step) 
$$\begin{array}{c} X, \underline{C}_1 \vdash D, \dots, X, \underline{C}_n \vdash D \\ \hline X \vdash D \end{array}$$

(%)

$X$  a finite set of facts (= closed atoms)

$X \downarrow D$  iff  $D = \dots \vee (E \mathbf{x}) C \vee \dots$  and  $X$  contains all facts  
in  $C[\mathbf{x}:=\mathbf{a}]$  for suitable parameters  $\mathbf{a}$

(%)  $C \Rightarrow D$  a *closed* instance of axiom in  $T$  with  $C$  included in  $X$  and  
 $D = \dots \vee (E \mathbf{x}) C_i \vee \dots$  and each  $\underline{C}_i$  a *fresh* instance of  $C_i$  ( $1 \leq i \leq n$ )

# Metaproperties

- Soundness
- Completeness
- Constructivity
- Conservativity
- Undecidability
- Automation (SATCHMO!)

# Examples

- exist.in
- or.in
- nijm.in

# Case studies

- Confluence theory: induction steps in Newman's Lemma, Hindley-Rosen, Self-lengthening Thm, ..
- Lattice theory:  $x \cap (y \cup z) \leq (x \cap y) \cup (x \cap z)$  for all  $x, y, z$  implies  $(x \cup y) \cap (x \cup z) \leq x \cup (y \cap z)$  for all  $x, y, z$
- Projective geometry: equivalence of two versions of Pappus' Axiom (1 minute, 1MB proof)